APPENDIX

[to The Selective Electrostatic Storage Tube
RCA REVIEW
March 1951, Vol.XII, No. 1
By J. Rajchman]

THEORY OF CONNECTIONS OF PARALLEL BARS FOR COMBINATORIAL SELECTION

[BY GEORGE W. BROWN]

The selective storage tube uses, for area selection, a matrix of control electrodes which stop the electrons from a uniform electronic bombardment in all but a single location, or a predetermined number of locations, rather than deflecting an electron beam to the chosen location as is done in conventional cathode ray tubes. It is obvious that such a go, no-go control at positions materially fixed by the matrix provides a greater certainty of selection than is possible by controlling precisely the amplitudes of deflection of a beam. In addition, several or all locations may be selected simultaneously. These advantages are obtained at the expense of a large matrix of control electrodes. It turns out, however, that these electrodes may be simple straight parallel bars, which can be connected into a number of groups so relatively small that the system is practical even when the number of selected elements is very large, e.g., a few millions.

The system is based on the idea that the space between two adjacent bars can be considered as a gate for the passage of electrons, which is opened only when both limiting bars are at some relatively more positive potential, and closed when at least one is at a sufficiently more negative potential. Consider a loop of N parallel bars, separated by N spaces or gates, like the pickets of a closed fence. Because of the dual coincident control on each gate by its limiting bars, it should be possible to connect the *N* bars into *G* groups to produce gate selection by making two selected groups positive and the remaining negative, in such a way that *N* be proportional to G^2. This possibility would be obvious if the two bars controlling each electron gate did not affect also the two adjacent gates, as for example, the two control grids of a converter tube. It is a purpose of this appendix to show that the row of *N* gates formed by *N* bars can, in fact, be connected as economically as the *2N* control elements of a row of *N* independent coincident devices in spite of the restriction imposed by the fact that there is a "fixed connection" of the two halves of each bar which connects necessarily adjacent gates. This restriction limits only the choice of the geometrical location of the gates with respect to the group codes to which they belong, and is irrelevant in storage devices for handling coded information, since identification of each element is by its selecting bars only and the "picture" formed by the elements plays no role.

First consider the simple case of a row of *N* coincident double grid tubes, where N is not prime:

$$N = P \bullet Q \qquad (1)$$

As coincidence is similar to the intersection of lines, it is convenient to think of all the grids grouped into one connection as lying on a line, the intersections of the lines being the coincident devices. The G groups of grids may be divided into two families of *P* and Q, represented by a set of *P* lines intersecting a set of *Q* lines, as on the symbolic diagram of Figure 22, each intersection corresponding to one tube.

$$G = P + Q \quad (2)$$

Since each *P* (and each Q) group has *Q* (and *P)* grids, the *2N* grids are accounted for. For a square pattern, *P=Q = G/2,* and *N* = G-/4. Instead of selecting the two groups to determine a tube, one from the *P* and one from the Q family, the two can be chosen in more ways among the whole set *G,* provided that:

$$N = \frac{G(G+1)}{2} \quad (3)$$

FIGURE 22.

This is illustrated in Figure 22 where each of the G lines intersects all remaining (G - 1) lines. This one-family system is more economical than the two-family system, since *N* tends to $G^2/2$ for large G's. In the case of either the one- or the two-family system, the *N* independent devices may be permuted arbitrarily on the A^2 intersections so that there are *N!* possible different systems of connections. In particular, the two-family system can be connected so that the selected element progresses monotonically (shown in Figure 22) along the row as the values of *p* are successively 0, 1, 2, ... *p,* (p- l), for *q* = 0 and again through the cycle 0 to *(p — l)* for *q* = 1, etc., up to *q = Q ~ 1,* so that the ordinal number *n* of the selected element is

$$n = P \cdot q + p. \quad (4)$$

FIGURE 23.

Consider now the electron gates formed by the bars. In a similar symbolic diagram the lines represent again the connections between bars and the intersections represent the gates. However, the bars are not on the intersections as were the grids, but between them. Typical connections of 16 bars *(P = Q* = 4, *N* = 16) for a two-family system are shown in Figure 23. A general method for finding a two-family connection system is as follows: Start at any intersection out of the *N = P Q* intersection formed by the *P* and *Q* lines, call it gate *n* = 0 (for example, *p* = l, *q* = l, Figure 23). Then along the *q* line (or *p*) containing gate 0, choose any intersection *p* (or *q)* for gate n = 1 (e.g., *p = 2*, *q* = 1, *n* = 1*)*. The first *q* chosen (*q* = 1 in the example) is the group number of the bar separating gates *n* = 0 and *n*= 1. Since gate *t i* = 1, as any gate, must be limited by a *q* and a *p* bar, proceed now on the *p* line containing gate 1 and choose any intersection for gate 2 (q = 3, p = 2, *n = 2)*. Continue similarly for gates 3, 4 ... up to *(n* — 1), alternately moving along *Q* and *P* lines in such a way as to never choose an intersection already numbered. There are many trial routes which will end up by filling the whole matrix. Many others will end in an impasse on a filled *P* or *Q* line, there being still vacant points elsewhere. It will be shown now that if *P* and *Q* are even, there is at least one successful route or connection system such that any group in the *P* (or *Q)* family has a bar adjacent to a bar of every group in the *Q* (or *P*) family, once and once only.

The N bars may be numbered by the same numbers $n = 0, 1, 2, 3 \ldots (n - 1)$ used for the gates, with a chosen sense of rotation. The bars with even n's will be assigned in succession to the groups of index p [$p = 0, 1, 2, \ldots$ to $(P - 1)$] as follows: $n = 0$ to $p = 0$, $n = 2$ to $p = 1$, $n = 4$ to $p = 2$, $n = 2p$ to p, up to $n = 2P - 2$ to $p = P - 1$. This cyclic assignment is done $Q\ 2$ times, creating P groups containing Q/2 bars each, accounting thereby for $PQ\ 2$ even numbered bars. This procedure can be expressed by the formula:

$$p = ½R_{2P}(n) \text{ when } n \text{ is even or } [R_2(n) = 0] \qquad (5)$$

where the notation $R_a(b)$ is used for the remainder of the division of b by a. The odd n's will now be assigned to the Q groups as follows:

To $n = 1$ assign q = 0; to $n = 3$, Q = 1; to n=5 assign Q = 0 again; and

to $n = 7$, q = 1 again etc.; . . . n to q = 0 when $R_2\left(\frac{n-1}{2}\right) = 0$ and

n to q = 1 when $R_2\left(\frac{n-1}{2}\right) = 1$ . . . up to $n = 2P — 1$ assigned to

q = 1. To the second $2P$ cycle of the n bars assign the pair q = 2 and q = 3, in a similar way: the bar $n = 2P + 1$ to $q = 2$; $n = 2P + 3$ to $q = 3$; $n = 2P + 5$ to 7 = 2; $n = 2P + 1$ to 7 = 8 etc.; . . . $n = 2P + R_{2p}(n)$ to 7 = 2 when $R_2\left(\frac{n-1}{2}\right) = 1$ and to q = 3 when

$R_2\left(\frac{n-1}{2}\right) = 1$, up to . . . n = 4P - 1 to g = 3. The third cycle will be connected similarly to the pair q = 4, 5, the fourth to q = 7, 8 etc., with the last pair being connected to $(Q - 2)$ and $(Q - 1)$. Each group Q contains $P/2$ bars, and the N bars are divided into $Q / 2$ cycles of $2P$ bars each. This accounts for the $P\ Q\ 2$ odd numbered bars. The assignment of the q bars can be expressed formally:

$$q = n - \frac{R_{2P}(n)}{P} + R_2\left(\frac{n-1}{2}\right) \quad \text{when } n \text{ is odd or } [R_{2},(n) = 1]. \tag{6}$$

It is evident that this system of connections satisfies the required conditions. Indeed, for any selected set of p and q, there is a cycle of $2P$ bars corresponding to the pair of q's to which the particular q belongs, since all values of q were assigned, and in that cycle, one of the bars of the selected group must be adjacent to a bar of the selected p group, since the bars of the q group are adjacent to a bar of all p groups.

Figure 24 shows an example of the two-group family system, according to the teachings of the above proof, for P = 8, Q = 4, G = 12, N = 32, obtained by applying relations (4) and (5) for all values of n. This particular system corresponds to the zigzag line in the symbolic diagram which orders the gates as closely as possible to the monotonic order of Figure 22, obtained with the independent double gate system. A completely monotonic order is impossible since there must be alternate P and Q segments. There are, of course, many other possible combinations. Just how many less than the $N!$ connections, possible with the independent double control devices, are still possible with the row of bars, is an interesting problem of combinatorial analysis. This problem amounts to determining the number of ergodic paths in a mesh of points when walking is subject to the restriction of making consecutive steps in alternate directions.

When the bars are shown in a straight row, rather than a closed loop, the extreme bars must be considered adjacent (bars 0 and 31 on Figure 24). a condition realized in practice by an additional bar on one end connected to the bar on the other end. Such additional bars are necessary also when there are gaps in the row of gates. This is the case in the tube described, at the location of the central cathode-supports.

The economy in the number G of vacuum seals and external circuits of the two-family group system of N gates can be measured by the ratio of merit N / G or $P\,Q / P + Q$. This ratio is the greatest when $P - Q$ *is* the smallest. For example, when $N = 32$, $N / G = 2.67$ for $P = 8$ and $Q — 4$ and only $N / G = 1.78$ when $P = 16$ $Q = 2$. The most efficient system is, of course, when N is the square of an even number, then $P = Q$ and the ratio of merit is $P / 2$. It is obvious also that the advantage of the system of connections grows with its size.

FIGURE 24.

There are more ways to choose two groups out of G than one group out of each P and Q, where $G = P + Q$, as was mentioned in the case of independent coincident devices. It turns out, as with the two-family system, that the single-family of groups system is possible with the bars when G is odd and also that there is no loss of economy in the connections but merely a restriction on possible choices of location of the gates. The number of gates must be of the form of Equation (3). Figure 22 was drawn so as to illustrate the case of *N* bars (as well as a special case of *N* independent double coincident devices). A system of connections is obtained by drawing *G* lines, each line intersecting the *(G* — 1) others, and "walking" from any origin, so as to never take two consecutive steps on the same line. There are many, but less than *N!* such walks which pass through all points. A method certain to yield a successful connection system, when *G* is prime, is to make a list of the group numbers, *G* = 0, 1, 2 . . , *(G—l)*, according to the order of the bars belonging to them, as follows: Start with any group, e.g., 0, and make additions modulo *G,* first by adding 1 *G* times, then

FIGURE 25.

2 *G* times, then 3 *G* times, etc., up to $\frac{G-1}{2}$. All $\frac{G(G-1)}{2}$ bars will

have thus been numbered and each group is a neighbor with all others, once and once only. Figure 25 illustrates two examples of this procedure, one for *G* = 5 and one for *G = 7.* The second example is also illustrated on the diagram of Figure 22. The single family of groups system is more economical — asymptotically by a factor of two — than the two-family groups system as mentioned before. However, it is inconvenient -in most circuit applications in that the selection of one group influences the selection of the second. Area selection of elements is obtained by two systems of straight gates at an angle to each other through which electrons pass in series. the tube described in this paper, there are two systems of straight parallel bars normal to each other. It is possible also to form the selecting matrix with a coaxial pile of rings and equiangular straight bars parallel to the axis, or by two concentric sets of parallel helical bars rotating in opposite directions. In fact, such geometries were used in the early tubes. It is clear that the number of windows *E* created by the intersections of the gates is simply the product of the number of gates in each direction, while the total number of groups L, i.e., controlling leads, is the sum of the leads used in each direction, $L = G1 + G2$. For the case of the two square two-family systems of $N = G^2/4$ gates each, the total number of windows is:

$$E = \left(\frac{L}{4}\right)^4 \tag{7}$$

The merit ratio between the number of windows or selected elements and controlling leads is, therefore, $L^3/256$, showing that the advantage grows tremendously with size. For $L = 16$, *E* is 256 but for $L = 128$ the number of elements, $E = 1{,}048{,}576$, surpasses a million. Similar compound formulas may be derived for the single family system.

The fourth power relation comes about naturally from the four bars limiting each window. Each electron passageway is equivalent to a four-control grid tube. Of course, to a higher number of control electrodes corresponds a still higher power relation. With *n* control electrodes for each electron channel, the relation between the number *E* of selectable elements and the control leads *L* is:

$$E = \left(\frac{L}{n}\right)^n \tag{8}$$

Successive rows of aligned parallel bars have the particular restriction to the combinations of connections mentioned before, arising from the fact that each bar is adjacent to two gates. It turns out that, in spite of this, connections yielding the full advantage according to relation (8) are possible. The only limitation is again in the choice of the geometrical location of the gates which is irrelevant in storage tubes. As an illustration, consider the case of the binary connection system, where

$$E = 2^n$$

elements are controlled by *n* pairs of control leads

$$L = 2n.$$

Each pair is one push-pull input, one or the other lead being the more positive. Figure 26 shows an example of 64 gates controlled by three successive rows of bars. Two binary inputs, or two pairs, are assigned to each row, so that in each row 1/4 of the gates are open and 1/8 closed. The opened quarters in successive rows are interlaced so as to leave only one open channel. On the example of Figure 26, the inner row provides a division in geometrical halves with the control of one pair and opposed quarters with the control of the next pair. The middle row again divides geometrically the gates into opposed eighths and sixteenths. However, in the last row such ordered geometrical division is not possible, because each bar is necessarily a neighbor to two gates. In fact, this is the only restriction to the obvious way of pyramiding control grids in halves, quarters, eighths, etc.

FIGURE 26.

A binary address for the information is used often in a storage device in computing or information-handling machines, because of its inherent economy. A tube with direct binary address requires half as many successive rows of bars as there are digits. For a fairly large capacity, this number of rows may be appreciable. Instead, the tube can be made with a two-family group system, the numbers of groups *P* and Q being powers of two.* External circuit matrices are used to convert the binary address into the appropriate bases. This is done in the tube described in this paper. Such external converting matrices may be common to many tubes in parallel providing simultaneous access to identical addresses in all tubes. For this reason the overall economy of tube and circuits favor the simpler tubes with the group-of-two connection systems.

Several windows may be opened simultaneously by the simple expedient of connecting several groups together. This is equivalent to several smaller tubes connected in parallel and requires separate input and output channels for each separate region within the tube. It is also possible to open all windows simultaneously by making all bars positive as is done for information holding purposes as described above.

- G. W. Brown was the first to suggest the one and two family systems after analyzing the author's original conception of a purely binary system See U. S. Patents 2,494,670 and 2,519,172.